



CONFIDENTIAL

50X1-HUM

or grid circuits. In the former case a negative grid bias, varying automatically with variation in oscillator rhythm, is obtained on account of the emission current of an electronic tube (Figures 1 and 2); in the latter, on account of grid currents flowing through a grid leak. Such automatic-bias circuits are used in an overwhelming majority of vacuum-tube oscillator systems to develop the necessary grid bias without using special batteries and to improve oscillator stability.

The phenomenon of self-modulation takes place when the operations carried out are unstable without automatic bias and the capacity of an automatic-bias circuit exceeds a certain critical amount. This means that a vacuum-tube oscillator, instead of "normal" quasi-sinusoidal self-excitation, produces self-excitation modulated according to amplitude (the form of self-modulation and its period are determined by the parameters of the vacuum-tube oscillator). As the capacity of an automatic-bias circuit increases, the self-modulation of self-excitation increases in depth and is gradually converted into intermittent oscillation expressed by periodic cessations of high-frequency oscillations.

This article studies a self-excited vacuum-tube oscillator with automatic cathode-bias circuit, by assuming that such an oscillator approximates a linear oscillatory conservative system and by using so-called abridged equations.

#### B. Abridged Equations

Let us examine a vacuum-tube oscillator with automatic cathode-bias circuit (Figure 1). The processes in such an oscillator are determined by the following system of differential equations (see notations in Figure 1)

$$\left. \begin{aligned} L \frac{di}{dt} + Ri &= v - M \frac{di_a}{dt} \\ v_g &= v + u \\ -C \frac{dv}{dt} &= i + i_g \\ -C_k \frac{du}{dt} &= i_a + i_g + \frac{u}{R_k} \end{aligned} \right\} \quad (1)$$

Let us introduce abstract dimensionless time:

$$\tau = \omega_0 t, \quad (2)$$

where  $\omega_0 = \frac{1}{\sqrt{LC}}$  is the natural frequency of the oscillatory circuit; and the dimensionless dependent variables  $x, y, z$  are defined as follows:

$$v = ax, \quad i = -\frac{a}{\omega_0 L} y, \quad v_g = az \quad (3)$$

( $a$  is some unit of voltage).

Let us assume that the anode and grid currents of the electronic tube do not depend on the anode voltage  $v_a$ , but are simple and derivative functions of the grid voltage  $v_g$ :

$$i_a = i_a(v_g), \quad i_g = i_g(v_g). \quad (3a)$$

Then, when  $i_g \ll i_a$ , the system of equations (1) is reduced to a system of the following dimensionless form:

$$\left. \begin{aligned} \frac{dx}{d\tau} &= y - \kappa \gamma f(z) \\ \frac{dy}{d\tau} &= -x - \mu y + \mu \alpha f'(z) \frac{dz}{d\tau} \\ \frac{dz}{d\tau} - \frac{dx}{d\tau} &= -\frac{\mu}{2\beta} [z - x + q f(z)] \end{aligned} \right\} \quad (4)$$

CONFIDENTIAL

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

where

$$f(z) = \frac{1}{aS} i_a(az), f_g(z) = \frac{1}{aS_g} i_g(az) \quad (5)$$

are reduced to the dimensionless form of electronic tube characteristics ( $S$  and  $S_g$  must denote the sharpness of the characteristics for anode and grid currents, respectively, at any fixed point, for instance, in the linear parts of the characteristics);

$$\mu = \omega_0 RC = \frac{R}{\omega_0 L} \quad (6)$$

is the attenuation of the oscillatory circuit, and:

$$\alpha = \frac{MS}{RC}, \beta = \frac{1}{2} \omega_0 RC \omega_0 R_k C_k^*, \gamma = \frac{LS_g}{RC} \text{ and } q = SR_k^{**} \quad (7)$$

\*(The parameter  $\beta$ , which can be written  $\beta = R_k C_k \frac{\omega_0^2 L}{R}$ , is nothing but the ratio of the time constants of the automatic-bias circuit  $R_k C_k$  and the oscillatory circuit  $\frac{2L}{R}$ .)

\*\* (If the electronic vacuum-tube oscillator is a tetrode or pentode, the screen current  $i_g$  will also flow through the automatic-bias circuit. Then, assuming proportionality between the anode and screen currents, we obtain  $q = (S + S_g) R_k$ , where  $S_g$  is the sharpness of the screen current characteristic.)

If the nonlinear system under discussion approximates the conservative system, fairly satisfactory results can be obtained by the van der Pol method; that is, by substituting for the former system of differential equations a simple nonlinear system of so-called abridged equations [8 - 11].

To pass from the system of equations (4) to the abridged system we shall assume that the positive parameter  $\mu$  is small and that the quantities  $\alpha, \beta, \gamma$  and  $q$  are of the order of unity so that when  $\mu \rightarrow 0, \mu\alpha, \mu\beta, \frac{\mu}{q}$  and  $\frac{\mu}{q} q$  then toward zero.

When  $\mu = 0$ , the solution of system (4) (now linear and conservative) is

$$x = V \cos(\tau + \varphi) \quad y = -V \sin(\tau + \varphi) \quad z = x + U, \quad (8)$$

where  $V, U, \varphi$  are arbitrary constants.

In equation system (4) (when  $\mu \neq 0$ ) let us substitute  $V, U$  and  $\varphi$  for  $x, y$  and  $z$  in accordance with (8). Substituting (8) in (4) and solving for  $\frac{dV}{d\tau}, \frac{dU}{d\tau}, \frac{d\varphi}{d\tau}$ , we obtain a non-autonomous system defining  $V, U, \varphi$  as slowly varying functions of time  $\tau$  (their rates of variation are of the order of the small parameter  $\mu$ ). The abridged equation system is obtained by eliminating the right-hand terms in  $\tau$  and by discarding terms in  $\mu$  higher than the first power. Obviously, it will now be written in the form

$$\left. \begin{aligned} \frac{dV}{d\tau} &= \beta V \{-1 + \alpha \Phi_1(U, V) - \gamma \Phi_2(U, V)\} \\ \frac{dU}{d\tau} &= -U - q \Phi_2(U, V) \end{aligned} \right\} \quad (9)$$

- 3 -

CONFIDENTIAL

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

(The right-hand terms of the first equation in (9) have an obvious physical significance. The first term,  $\beta V$ , reflects the attenuation of oscillation because of oscillating current losses; the second reflects the increase in oscillation amplitude because of the presence of an electronic tube, and feedback; the last reflects the decrease in amplitude because of grid circuit losses caused by grid circuit reaction to the oscillating circuit.)

where

$$\tau' = \frac{\mu}{2\beta} \quad \tau = \frac{t}{R_k C_k} \quad (10)$$

is "slow" time and

$$\left. \begin{aligned} \Phi_1(U, V) &= \frac{2}{\pi} \int_0^\pi f'(U + V \cos \xi) \sin^2 \xi d\xi \\ \Phi_{1g}(U, V) &= \frac{2}{\pi} \int_0^\pi f'_g(U + V \cos \xi) \sin^2 \xi d\xi \\ \Phi_2(U, V) &= \frac{1}{\pi} \int_0^\pi f(U + V \cos \xi) d\xi \end{aligned} \right\} \quad (11)$$

The "shorter" equation for  $\frac{d\varphi}{dt}$  gives  $\varphi \equiv$  a constant, that is, the correction for frequency of the order  $\mu$  equals zero.

If the oscillating current is in the anode circuit (Figure 2), the same abridged equations are obtained for  $V$ , the amplitude of the variable component, and for  $U$ , the constant component of the grid voltage, but with

$$\gamma = \frac{L_g S_g}{RC} \quad (12)$$

For further analysis of equations (9), let us approximate the characteristics of the anode and grid currents of the tube by means of the discontinuous linear functions following:

$$i_a = S(v_g - v_g^0) I(v_g - v_g^0), \quad i_g = S_g v_g I(v_g) \quad (13)$$

(Figure 3), where the unitary function, as usual, is

$$I(z) = \begin{cases} 0 & \text{when } z < 0, \\ 1 & \text{when } z > 0. \end{cases} \quad (13a)$$

Taking

$$a = -v_g^0 > 0, \quad (14)$$

as the unit of voltage, we shall obtain

$$f(z) = (z+1) I(z+1), \quad f_g(z) = z I(z) \quad (15)$$

and abridged equations in the form

$$\left. \begin{aligned} \frac{dV}{d\tau} &= P(V, U) = \beta V \{-1 + \alpha \psi_1(\gamma_2) - \gamma \psi_1(\gamma_1)\} \\ \frac{dU}{d\tau} &= Q(V, U) = -U - \gamma V \psi_2(\gamma_2) = \\ &= -U - \frac{\gamma}{\pi} \{(U+1)\theta_2 + V \sin \theta_2\} \end{aligned} \right\} \quad (16)$$

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

where  $\gamma_1 = \frac{U}{V}$ ,  $\gamma_2 = \frac{U+1}{V}$  and  $\theta, \theta_2$  are determined in accordance with  $\gamma_1, \gamma_2$  by the functions

$$\theta = \theta(\gamma) = \begin{cases} 0 & \text{when } \gamma < -1 \\ \arccos(-\gamma) & \text{when } |\gamma| \leq 1 \\ \pi & \text{when } \gamma > 1 \end{cases} \quad (17)$$

and

$$\varphi_1(\gamma) = \frac{1}{\pi} [\theta(\gamma) + \gamma \sin \theta(\gamma)], \quad \varphi_2(\gamma) = \frac{1}{\pi} [\sin \theta(\gamma) + \gamma \theta(\gamma)] \quad (18)$$

### C. Singular Points

Let us examine the following phase semiplane of abridged equation system (9):

$$-\infty < U < +\infty, \quad V \geq 0.$$

The singular points in the phase plane, points corresponding to oscillator processes with the constants  $U$  and  $V$ , are defined by the system of equations  $P(V, U) = 0$  and  $Q(V, U) = 0$ .

There is always a singular point  $(U_0, 0)$ , on the  $U$ -axis ( $V = 0$ ), corresponding to the unexcited state of the oscillator. The value of  $U_0$  (on the abscissa axis) is determined from the relation  $Q(0, U_0) = 0$ ; that is, it equals

$$U_0 = \frac{-q}{1+q} \quad (19)$$

It is easy to show that this singular point is a stable node if  $\alpha < 1$ , and a saddle node (metastable point) if  $\alpha > 1$ .

When  $\alpha < 1$  outside the axis  $V = 0$ , there are no singular points and therefore there are also no limiting periods and all phase traces converge, when  $t \rightarrow +\infty$ , to the stable node  $(U_0, 0)$ .

When  $\alpha > 1$  there is a single (for each combination of the parameters  $\alpha, \gamma, q$ ) singular point  $(\bar{U}, \bar{V})$  outside the axis  $V = 0$  -- a point corresponding to periodical (unmodulated) self-excitations. In the given scale,  $\bar{V}$  is the amplitude of self-excitation and  $\bar{U}$  is the constant component of voltage in the automatic-bias current. This point is defined as the point of intersection of the following two curves: the R-curve

$$R(V, U) = \alpha \varphi_1(\gamma_2) - \gamma \varphi_1(\gamma_1) - 1 = 0 \quad (20)$$

and the Q-curve

$$Q(V, U) = -U - qV\varphi_2(\gamma_2) = 0. \quad (21)$$

(The reason for there being but one singular point is that along the Q-curve, the parameter  $\alpha$  of the R-curve is a monotonically increasing function of  $V$ .)

Figure 4 gives the family of R-curves (parameters of the family  $\alpha$ ) for  $\gamma = 20$ ; other  $\gamma$ 's have analogous families. The larger  $\gamma$  is (as compared with  $\alpha$ ), the nearer the R-curves in the region  $\gamma_1 > -1$  become a straight line  $\gamma_1 = -1$ . It should be noted that in the region  $\gamma_1 < -1$  the R-curves are a set of straight lines in which  $\bar{\gamma}_2$  is determined by the relation:

$$\left. \begin{aligned} U &= -1 + \bar{\gamma}_2 V \\ \alpha \varphi_1(\bar{\gamma}_2) &= 1 \end{aligned} \right\} \quad (22)$$

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

Figure 5 illustrates a family of Q-curves. Each of the Q-curves begins at a corresponding point  $(U_0, 0)$ .

It should be observed that the R- and Q-curve families are "Reiss" diagrams [4], since they express the dependence of the self-excitation amplitude  $V$  on the constant component of the grid voltage  $U$ , the former, for constant feed-back ( $\alpha = \text{a constant}$ ) and the latter, for constant resistance of automatic-bias circuit ( $q = \text{a constant}$ ).

The stability of a singular point  $(\bar{U}, \bar{V})$ , i.e., the stability of corresponding periodic (unmodulated) self-excitation (with  $U$  and  $V$  equal to constants), is defined, as we know, by the expression

$$D_1 = \frac{\partial P}{\partial V} + \frac{\partial Q}{\partial U} \text{ and } D_2 = \frac{\partial(P, Q)}{\partial(V, U)}$$

at a singular point. In accordance with (16)

$$D_1 = \frac{\beta}{\pi} (\alpha \sin 2\bar{\theta}_2 - \gamma \sin 2\bar{\theta}_1) - \left(1 + \frac{q}{\pi} \bar{\theta}_2\right) \quad (23)$$

and

$$D_2 = \frac{2\beta}{\pi} (\alpha \bar{\theta}_2 \sin \bar{\theta}_2 - \gamma \bar{\theta}_1 \sin \bar{\theta}_1) \left(1 + \frac{q}{\pi} \bar{\theta}_2\right) + \frac{q}{\pi} \sin \bar{\theta}_2 (\alpha \sin \bar{\theta}_2 - \gamma \sin \bar{\theta}_1) = \\ = \frac{\alpha}{V} \sin \bar{\theta}_2 + \frac{\gamma \sin \bar{\theta}_1}{V} \frac{q}{\pi} \bar{\theta}_2 > 0, \quad (24)$$

(Lines over the symbols indicate values of these magnitudes at the singular point  $(\bar{U}, \bar{V})$ .) In other words, the singular point  $(\bar{U}, \bar{V})$  is a node or focus, stable when  $D_1 < 0$  and unstable when  $D_1 > 0$ . If the singular point  $(\bar{U}, \bar{V})$  lies in the region  $\alpha \sin 2\bar{\theta}_2 - \gamma \sin 2\bar{\theta}_1 < 0$  in particular, this will be when  $\alpha < 2$  or when  $q < \pi$ , it is stable for any  $\beta$ . But if the singular point  $(\bar{U}, \bar{V})$  lies in the region  $\alpha \sin 2\bar{\theta}_2 - \gamma \sin 2\bar{\theta}_1 > 0$ , i.e., in the labile area (L) (Figure 4) between the geometric site of the (s)-points of the R-curves, with vertical tangents, the straight line  $\gamma_2 = -1$  and the part of the straight line  $u = -1, 0 < v < 1$ , then the singular points  $(\bar{U}, \bar{V})$  are stable when  $\beta < \beta_0$  and unstable when  $\beta > \beta_0$ , where the "bifurcated" value of the parameter  $\beta$  is

$$\beta_0 = \beta_0(\alpha, \gamma, q) = \frac{1 + \frac{q}{\pi} \bar{\theta}_2}{\frac{\alpha}{\pi} \sin 2\bar{\theta}_2 - \frac{\gamma}{\pi} \sin 2\bar{\theta}_1}. \quad (25)$$

The corresponding critical value of the capacity of the automatic-bias circuit will be

$$(C_k)_{\text{critical}} = C^0 \frac{\beta_0}{q}, \text{ where } C^0 = 2 \frac{S + S(q)}{\omega_0 \mu}. \quad (26)$$

It should be noted that the labile area (L) corresponds to the working conditions of self-excitation, which is unstable without automatic bias (constant grid bias  $U$  is generated by a special battery), and the singular point  $(\bar{U}, \bar{V})$  lies in the labile area (L) when

$$q > \pi \text{ and } 2 < \alpha < \alpha_0, \quad (27)$$

- 6 -

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

where  $\alpha_0 = \alpha_0(q, \gamma)$  is determined by the relations:

$$\left. \begin{aligned} \omega_0 \sin 2\theta_2 - \gamma \sin 2\theta_1 &= 0 \\ \alpha_0 \theta_2 - \gamma \theta_1 &= \pi \\ \cos \theta_1 - q \gamma_2 (-\cos \theta_2) &= 0 \end{aligned} \right\} \quad (27a)$$

#### D. Limiting Cycles. Self-Modulation

Research on the character of "bifurcation" when  $\beta = \beta_0$ , conducted in accordance with the work of Andronov and Leontovich [8] and Bautin [12] on the nature of a limiting cycle from a singular focus-type point showed that in the transition of a monotonically increasing parameter  $\beta$  through the "bifurcated" value  $\beta = \beta_0$  from the stable focus  $(\bar{U}, \bar{V})$  one stable limiting cycle appeared while the focus itself became unstable. This limiting cycle in van der Pol's plane obviously corresponds to self-modulation of self-excitation. When  $\beta \rightarrow \beta_0$  (but  $\beta > \beta_0$ ) the dimensions of the cycle converge to zero. When  $\beta$  approaches  $\beta_0$  (but  $\beta > \beta_0$ ) and the form of the limiting cycle is nearly ellipsoidal, then self-modulation is almost sinusoidal with frequency:

$$\Omega = \frac{\omega_0 k}{2\theta_0} \quad (28)$$

Furthermore, as may easily be demonstrated, the infinitely remote parts of the phase semiplane  $(\bar{U}, \bar{V})$  are absolutely unstable: all phase traces enter some finite area near the origin of the coordinates. Hence, when  $\beta > \beta_0$  in this area there is known to be at least one stable limiting cycle. By means of graphic integration of differential equations (16) for particular values of the parameters  $\alpha, \gamma, q, \beta$ , it is possible to show that: (1) when  $\frac{1}{\beta} > \frac{1}{\beta_0}$  and the singular point  $(\bar{U}, \bar{V})$  is stable, there are no limiting cycles in the phase semiplane; and that (2) when  $\frac{1}{\beta} < \frac{1}{\beta_0}$  and the singular point  $(\bar{U}, \bar{V})$  is unstable, there is one and only one limiting cycle in the phase semiplane and it is stable.

Thus, under any initial conditions, "normal" unmodulated self-excitations are set up in the vacuum-tube oscillator under consideration, if a process of self-excitation is carried out which is stable without automatic bias (singular point  $(\bar{U}, \bar{V})$  lying outside the L-area) or when conditions (27) are satisfied, but  $C_k > (C_k)_{critical}$ . When conditions (27) are fulfilled and  $C_k > (C_k)_{critical}$  is the oscillator, under any initial conditions, self-excitation modulated according to amplitude is established. When  $C_k = (C_k)_{critical}$  the result is continuous transition of the unmodulated self-excitation process to a self-modulated process (when  $C_k$  increases) and back (when  $C_k$  decreases); the extent of the self-modulation of self excitation is a continuous function of  $C_k$  and can be made as small as desired by selecting the corresponding capacity of  $C_k$ , sufficiently close to  $(C_k)_{critical}$ .

Such a process of developing self-modulation can be called soft (by analogy with the soft process of the development of self-excitation in a vacuum-tube oscillator) and is distinguished in its essential form from the hard process of the development of self-modulation discovered by Yevtanov [6] in the case of a vacuum-tube oscillator with a grid leak, and experimentally corroborated by the author.

With increase in  $\beta$  (after transition through  $\beta_0$ ), the dimensions of the limiting cycle in the phase semiplane increase and its form differs always more and more from the ellipsoidal and the lower part of the limiting cycle approaches more and more closely to the U-axis ( $V=0$ ). In other words, when the capacity  $C_k$  of the automatic-bias circuit increases, the result is a smooth transition from sinusoidal self-modulation, with modulation as small as desired, to intermittent oscillation which is expressed in periodically discontinuous high-frequency self-excitations and occurs when  $C_k \gg (C_k)_{critical}$  (when  $\beta \rightarrow +\infty$ ).

CONFIDENTIAL

CONFIDENTIAL  
CONFIDENTIAL

50X1-HUM

To illustrate the character of the "bifurcation" when  $\beta = \beta_0$  and the transition of quasi-sinusoidal self-modulation in intermittent oscillation with increase in  $\beta$ , Figures 6, 7, 8 and 9 give part of the phase pictures of the vacuum-tube oscillator under discussion, plotted from the results of graphic integration of equations (16) for  $\alpha = 4$ ,  $\gamma = 20$ ,  $q = 10$ ; and  $\beta = 4, 6, 15, +\infty$  ("bifurcated" value of the parameter  $\beta$  for given  $\alpha, \gamma, q$  is equal to  $\beta_0 = 5.0$ ). Figure 10 gives the curves showing the dependence of the V-amplitude of high-frequency self-excitations on the "slow" time  $\tau = \frac{1}{R_k C_k}$ , plotted in accordance with the corresponding limiting cycles in Figures 6 to 9. It is interesting to note that in the case under consideration the transition from quasi-sinusoidal self-modulation to intermittent oscillation occurs when

$$\frac{\beta}{\beta_0} = \frac{C_k}{(C_k)_{critical}} \sim 3$$

#### E. Extent of Parameters

A graphic representation of the process established in a vacuum-tube oscillator may be given by its parameters by expanding the oscillator parameters in various regions. The vacuum-tube oscillator in question has four essential positive parameters:

$$\alpha = \frac{M}{M_0}, \quad q = (S + S_g) R_k, \quad \frac{\beta}{q} = \frac{C_k}{C_0} \quad \text{and} \quad \gamma = \frac{L' S_g}{R_k C_k}$$

( $\frac{\beta}{q}$ , not  $\beta$  is taken as a third parameter, since the latter depends not only on  $C_k$  but also on  $R_k$ ), where  $M_0 = \frac{R_k}{C_k}$  is the imaginary induction coefficient  $M$  required for the self-excitation of an oscillator,  $C_0 = 2 \frac{S + S_g}{R_k}$ ,  $L' = L$  for an oscillating circuit in a grid circuit and  $L' = L_g$  for an oscillating circuit in an anode circuit.

For a given  $\gamma$  the surface  $\alpha = 1$  divides the area where self-excitation is absent (Figure 11, area I) from the area of "normal" unmodulated self-excitation (area II); the surface  $\frac{\beta}{q} = \frac{1}{q} \beta_0(\alpha, \gamma, q)$  that is,

$$\left. \begin{aligned} \frac{\beta}{q} &= \frac{\bar{\gamma}_1 + \bar{\gamma}_2}{2(\alpha \bar{\gamma}_2 - \gamma \bar{\gamma}_1 - \pi)} \\ \bar{\gamma}_1 + 4 \gamma \bar{\gamma}_2(\bar{\gamma}_2) &= 0 \\ \alpha \bar{\gamma}_1(\bar{\gamma}_2) - \gamma \bar{\gamma}_1(\bar{\gamma}_2) &= 1 \end{aligned} \right\} \quad (29)$$

divides area II and area III, the area of the self-modulation of self-excitation.

In Figure 11, cross sections of plane  $\alpha = 1$  and surface  $\frac{\beta}{q} = \frac{1}{q} \beta_0$ , for  $\gamma = 20$ , are given for the planes  $q = \text{const}$  ( $q = 5, 6, 8, 10, 20, +\infty$ ). For example, in this illustration the cross section of area III is crosshatched by the plane  $q = 10$ . Cross sections of these surfaces for other  $\gamma$ 's are identical with that shown in the illustration on the left-hand boundary of area III and differ only on the right-hand boundary corresponding to processes with grid currents.  $\frac{\beta}{q}$  does not depend on  $\gamma$  when  $\bar{\gamma}_1 < -1$  and when self-excitation is produced without grid currents; that is, when  $\alpha < \alpha_1$ , where  $\alpha_1 = \alpha_1(q)$  is determined by the following relations:

$$\alpha_1 = \frac{1}{\bar{\gamma}_1(\bar{\gamma}_2)}, \quad q = \frac{1}{\bar{\gamma}_2(\bar{\gamma}_2)} \quad (30)$$

In Figure 11, points  $(\alpha_1, q)$  are denoted by circles. The larger  $\gamma$  is, the nearer the cross section of the right-hand boundary of area III is to the straight line  $\alpha = \alpha_1(q)$ .

CONFIDENTIAL  
CONFIDENTIAL



CONFIDENTIAL

50X1-HUM

F. Experimental Results

The results obtained were checked on an oscillator, in connection with the plan of Figure 1, with a 6Zh7 [Zh=J 2] tube as an oscillator. In a very satisfying way its characteristics approximated the discontinuous linear functions in the form (13) with the following parameters:  $S=1.9 \mu A$ ,  $S_0=0.55 \mu A$ ,  $S_2=0.20 \mu A$ ,  $V_0=-10 V$  (for  $V_{g2}=250 V$  and  $V_{g1}=220 V$ ). The remaining parameters were as follows:  $L=97 mH$ ,  $C=500 pF$ ,  $[dF/dV]=\mu F/V$ ,  $\omega_0=1.4 \times 10^5/sec$ ,  $\mu=0.093$  and  $\gamma=65$ . Back-feed inductance  $M$  varied within the interval from 0 to 4 mH.

The measurements cited were completely corroborated not only qualitatively but quantitatively, within 10 to 15 percent of the calculated results of the theory stated above, for processes in a vacuum-tube oscillator with automatic cathode-bias circuit, both for amplitude of self-excitation without self-modulation and for critical capacity. A similar quantitative verification of theoretical results occurred with greater circuit attenuations (when  $\mu \sim 0.1$ ).

The "soft" process of the self-modulation phenomenon was demonstrated by a series of photographs of phase traces on the complex plane ( $U, V$ ), taken by means of a cathode oscillograph, which showed the genesis of a limiting cycle corresponding to "normal" unmodulated self-excitation when  $C_k$  passed through the critical value  $(C_k)_{critical}$ .

In contrast with the work of Bendikov and Gorelik [13], photographs were taken not only of singular points and limiting cycles, but also of phase traces which approximate them asymptotically. For this purpose voltage was supplied to one pair of deflecting plates from an automatic-bias circuit, and to the other was applied a voltage proportional to the "amplitude of self-excitation"  $V$ ; the latter was supplied from a diode detector connected with the oscillating circuit through a "buffer" cascade with initial resistance approximately  $10 M\Omega$ . To get the phase traces, corresponding with the transient processes, it was necessary repeatedly to fix the very same initial conditions in the oscillator. For this work a relay was used which short-circuited (about 15 times per second) the automatic-bias circuit.

Figures 12, 13, 14 and 15 [photographs, not reproduced] were obtained for the following values:  $M=1.4 mH$  ( $\alpha=8.2$ );  $R_k=8 k\Omega$  ( $q=20$ ); and capacitances  $C_k=0.086 \mu F$  (Figure 12),  $0.090 \mu F$  (Figure 13),  $0.092 \mu F$  (Figure 14), and  $0.095 \mu F$  (Figure 15). In this case  $(C_k)_{critical}=0.091 \mu F$  (calculation by formula (29) gives  $(C_k)_{critical}=0.095 \mu F$ ). In the first two photographs the capacity of the automatic-bias circuit was less than the critical capacity and the phase traces entered the stable focus ( $\bar{U}, \bar{V}$ ), corresponding to the steady-state process of unmodulated self-excitation. It can be readily seen how, when  $C_k$  approaches  $(C_k)_{critical}$ , the spacing of the spiral grows smaller and coils on ( $\bar{U}, \bar{V}$ ) and how, from the focus ( $\bar{U}, \bar{V}$ ) a stable limiting cycle makes its appearance (Figure 14).

Thus, a fully verified agreement between the results of theory and experiment shows that the "small parameter" method, the method of abridged equations, gives not only a qualitative but a quite satisfactory quantitative description of the processes of a vacuum-tube oscillator with automatic cathode-bias circuit (even with oscillating circuit attenuation  $\mu \sim 0.1$ ).

In conclusion, I consider it my duty to express my deep gratitude to Academician A. A. Andronov for his valuable advice and assistance.

CONFIDENTIAL

**CONFIDENTIAL**

50X1-HUM

## BIBLIOGRAPHY

1. Armstrong, E., Proc. IRE, 3, No 3, Sep 1915
2. Rzhavkin, S. N., Radio Technics, No 8, 1918
3. Rzhavkin, S. N., Vvedenskiy, B. A., TIT6P, 11, 67, 1921
4. Rukop, H., ZS. f. Techn. Phys., 441, 1924
5. Gorelik, G., Kuzovkin, V., Sekerskaya, Ye., "Techn. Radio i Sl. Toka," 629, 1932
6. Yertyanov, S. I., Electrocommunication, No 9, 66, 1940; No 11, 33, 1940
7. Barkhausen and Bose, High-Frequency Technics and Electroacoustics, 60, 37, 1942
8. Andronov, A. A., Khaykin, S. E., Theory of Oscillations, Ch. 1, ONTI, 1937
9. Mandel'shtam, L. I., Papaleksi, N. D. and others, Recent Research on Non-linear Oscillations, Gos. Izd. on Radio Problems, 1936
10. Bulgakov, B. V., Appl. Math. and Mech., VI, 396, 1942; I, 313, 1946
11. Bogolyubov, N., Some Statistical Methods in Mathematical Physics, Izd. AN SSSR, 1945
12. Bautin, N. N., ZhETF, 8, 759, 1938
13. Bendrikov, G. A., Gorelik, G. S., ZhTF, V, 620, 1935

[Appended figures follow.]

**CONFIDENTIAL**

CONFIDENTIAL

50X1-HUM

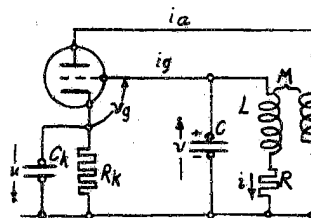


Figure 1

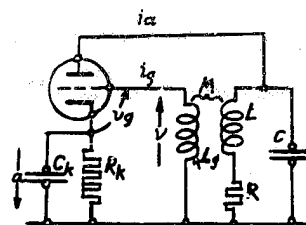


Figure 2

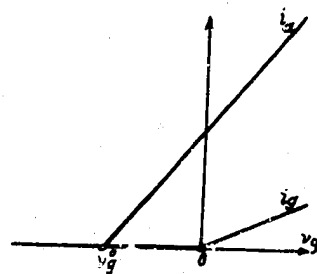


Figure 3

11 -

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

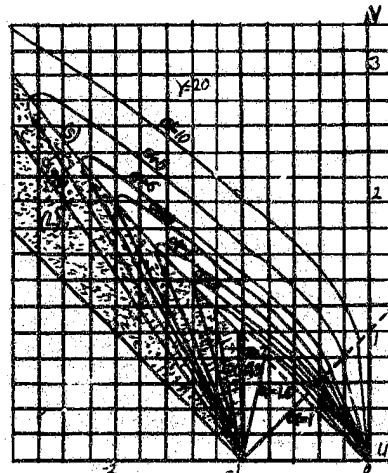


Figure 4

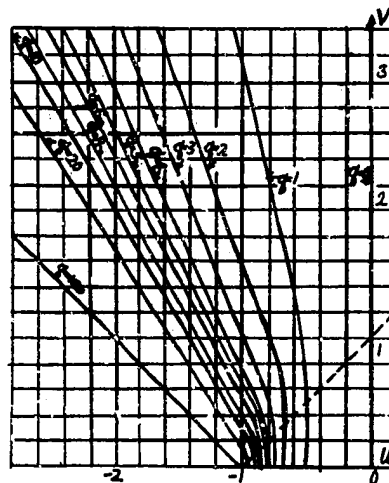


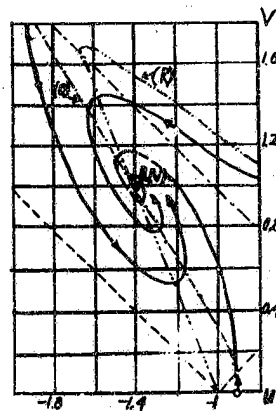
Figure 5

- 12 -

CONFIDENTIAL  
CONFIDENTIAL

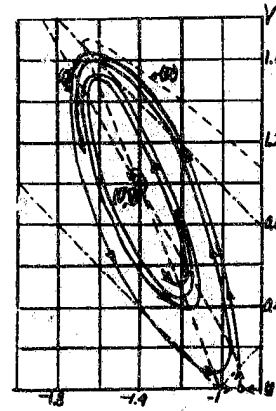
CONFIDENTIAL

50X1-HUM



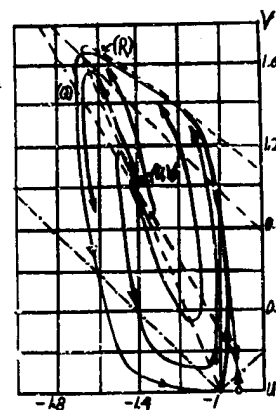
$$\alpha=4; \gamma=20; q=10; \beta=4.$$

Figure 6



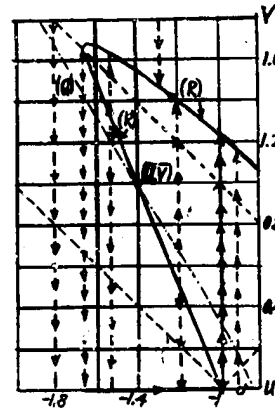
$$\alpha=4; \gamma=20; q=10; \beta=6.$$

Figure 7



$$\alpha=4; \gamma=20; q=10; \beta=1.5$$

Figure 8



$$\alpha=4; \gamma=20; q=10; \beta=+\infty$$

Figure 9

- 13 -

CONFIDENTIAL

**CONFIDENTIAL**

50X1-HUM

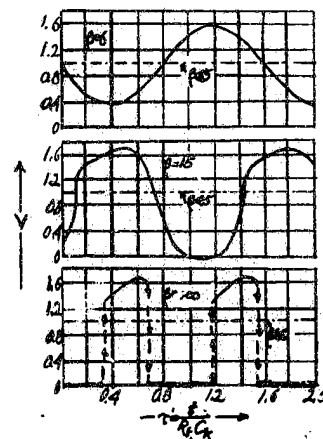


Figure 10

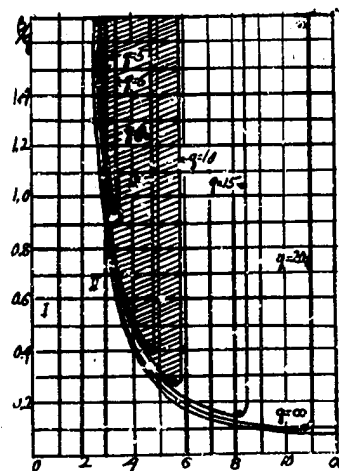


Figure 11

- E N D -

- 14 -

**CONFIDENTIAL**